Automated Game Analysis via Probabilistic Model Checking

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Abstract

This paper briefly reports on the analysis of a mixed-strategy game through probabilistic model-checking.

1 Introduction

It has been recognised for some time that there are close links between logics for the analysis of multi-agent systems and many game-theoretic models developed for the same purpose. In this paper, we contribute to this burgeoning body of work by showing how a probabilistic model checking tool can be used for the automated analysis of game-like multi-agent systems in which both agents and environments can act with uncertainty. Specifically, we show how a variation of the well-known alternating offers negotiation protocol of Rubinstein can be encoded as a model for the PRISM model checker [2]. In the variation we study, agents play mixed-strategy variations of several well-known negotiation tactics for the alternating offers model. We are able to automatically analyse properties of the model by expressing these properties in the probabilistic version of CTL [1].

2 Negotiation Framework

In the Rubinstein's negotiation framework [3] two agents, the *Buyer* (B) and the *Seller* (S), bargain over an item. Players alternatively take it in turns to make an action which can be either (i) throwing a proposal (offer), or (ii) accepting the most recent proposal. In theory both players can keep Rejecting offers so that an agreement may never be reached (in that case we talk about *disagreement* or *conflict deal*). However, a player's utility depends on the value at which an agreement is reached (as well as on the time at which it is reached), hence *disagreement* is the worst possible outcome for both players and it is ruled out.
Players' strategies (conceder/boulware): Players' strategies are determined by the so-called Negotiation Decision Function (NDF), which gives the value of a player's next offer. Strategies can be linear or non-linear, the latter being either *conceder*, if the player is willing to concede a lot in the early phase of negotiation, or *boulware* if a player is willing to concede considerably only when its time deadline is approaching. The following parameters are relevant for the characterisation of a player strategy. i)- $\boldsymbol{T^b}$ (time deadline): player **b** quits negotiation if an agreement has not been reached within T^b. ii)- $\boldsymbol{IP^b}$ (initial price): player **b**'s first offer. iii)- $\boldsymbol{RP^b}$ (Reserved Price): the threshold at which player **b** will certainly reject offer (hence also its maximum/minimum offered value, i.e. player **b** offers RP^b only at time T^b).

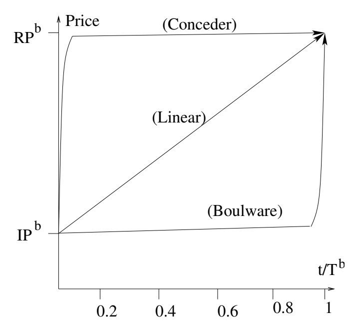

Figure 1: Buyer's Negotiation Decision Function

Acceptance probability: The probability for a player to accept a given offer depends on the offered value as well as on its RP. The following points are considered: i)- any offer below (above) the S_RP (B_RP) will be certainly rejected by the *Seller* (*Buyer*). ii)- the probability of accepting an offer (counter-offer) asymptotically tend to 1 as the offer (counter-offer) increases (decreases). iii)- for the sake of symmetry, players should have an equal attitude towards acceptance/rejection of an offer. The resulting *acceptance probability* functions, S_AP and B_AP respectively, are as follows:

$$S\_AP(x) = \begin{cases} 0 & \textbf{if } x <= S\_RP \\ 1 - \frac{S\_RP}{x} & \textbf{if } x > S\_RP \end{cases}$$

$$B\_AP(x) = \begin{cases} 1 & \textbf{if } x <= 0 \\ 1 + \frac{S\_RP}{x-(B\_RP+S\_RP)} & \textbf{if } 0 < x < B\_RP \\ 0 & \textbf{if } x > B\_RP \end{cases}$$

Figure 2 shows the curves for $S\_AP()$ and $B\_AP()$ with $S\_RP = 1000$ and $B\_RP = 10000$.

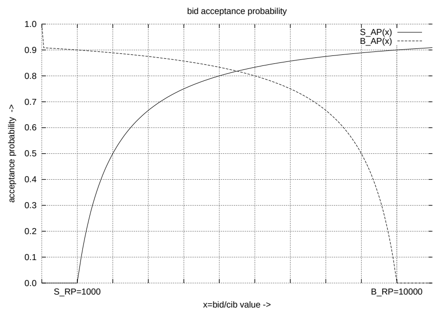

Figure 2: *Acceptance Probability* Functions

3 The Model

The negotiation framework has been modelled in PRISM as a DTMC consisting of two parallel modules, B and S, which synchronise on certain actions. The model is designed to cope with different negotiation strategies (linear, non-linear), and requires a suitable configuration (e.g. strategies' type and gradients, time-deadlines etc.) in order to be analysed.

4 Analysis

The probability an agreement is reached within the *Accepting Interval* is obtained by verification of the PCTL formula:

$$P = ?\ (true\, U\, ((s = 2) \& (b = 3) \& ((bid = PVAL) | (cbid = PVAL)))$$

which captures all the system's evolutions representing an agreement at value $PVAL$. A comparative analysis of negotiation strategies is achieved by running a number of experiments for $PVAL \in [S\_RP, B\_RP]$.

Linear strategies: Patience pays.
Our analysis shows that a player has more interest in adopting a *patient* behaviour (i.e. small increment, late deadline) rather than being impatient and quickly getting to offer his RP. In Figure 3, the Cumulative Probability of Agreement is compared for *linear* strategies with different gradients. Any curve corresponding to a *Seller* "slow tactic" (as in $Lin(x)$-$Lin(10)$) is "probabilistically" better than any one corresponding to a *Seller* "fast tactic" (as in $Lin(x)$-$Lin(100)$), as it concentrates most of the probability close to the *supremum* of the *Accepting interval*, independently of the *Buyer*'s behaviour.

Non-Linear Strategies: Boulware is better.
In our models, Non-Linear strategies are approximated via 2-segments-linear NDFs, consisting of a "slow", *boulware segment*, followed (preceded) by a "fast", *conceder segment*. A comparative analysis have shown that a player's probabilistic-profit increases with the duration of the *boulware segment*, independently of the opponent's strategy. This is pointed out by the curves in Figure 4 where *conceder-conceder* combination of strategies are compared: by shortening the Seller's *conceder* duration (while keeping the segments' gradients constant) the probability moves towards the interval *Supremum*.

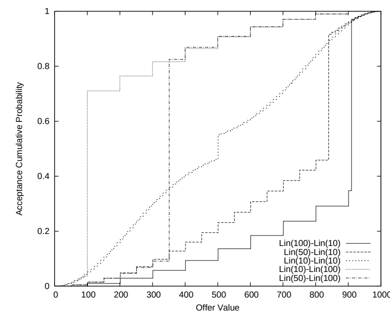

Figure 3: Cumulative Agreement Prob. for Lin-Lin strategies

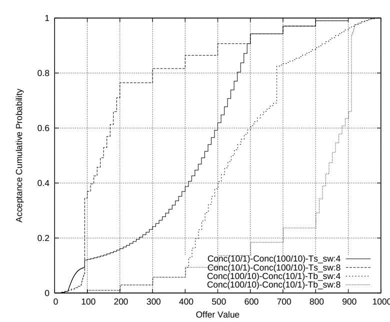

Figure 4: Cumulative Agreement Prob. for Conc-Conc strategies

References

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[3] M. J. Osborne and A. Rubinstein. *Bargaining and Markets*. The Academic Press: London, England, 1990.